Optical Design Tolerancing: A Key to Product Cost Reduction

During the design of any optical system destined to be manufactured, it is critical to define a fabrication and assembly budget. This budget must consider any potential compensation that will be used during the manufacturing process to mitigate the performance degradation introduced by fabrication variations. It is important to specify the best set of tolerances and compensators, as these will significantly impact the manufacturing costs. The complex process of defining system tolerances and compensators is often simply called, *tolerancing*.

Some minimum tolerances are dictated by the manufacturing process. It is important to perform a *sensitivity* analysis on these tolerances to determine the *as-built* performance of the system. Alternatively, a certain acceptable performance degradation may drive the tolerances. An *inverse sensitivity* analysis determines the set of tolerances that achieve a certain performance degradation. Both of these analyses should consider the effects of permissible compensators.

Why Is Tolerancing Important?

The simple answer is **cost**. A design may be very expensive to manufacture if small variations in the lens parameters result in significant performance loss even after compensation is applied. The ideal optical system design minimizes production costs *and* meets performance requirements using achievable component and assembly tolerances and well-chosen post-assembly adjustments. Optical design software can help to make this ideal manufactured system a reality, and the tolerancing process should be fast, flexible, and accurate.

Tolerancing with Optical Design Software

Many optical design software packages have the ability to perform a tolerance analysis; however, the algorithms used for tolerancing may differ. Ideally, a tolerancing algorithm provides a combination of **speed**, **accuracy**, and **insight** into which tolerances drive system performance; unfortunately, many optical design packages offer tolerancing algorithms that only provide two of these three.

CODE V is a comprehensive software package offered by Optical Research Associates (ORA$^{®}$) for the design, analysis, tolerancing, and fabrication support of optical systems. CODE V is used worldwide by organizations to design a wide range of optical systems for a variety of products, including digital camera equipment, medical instruments, aerospace systems, telecommunication components, microlithographic stepper systems, and many more. Many of these companies and users chose CODE V for its advanced algorithms and features related to tolerancing.

CODE V offers many traditional algorithms for calculating tolerances, and it also includes the **Wavefront Differential** tolerancing method that is extremely fast and accurate. The speed of this method enables tolerancing to become part of the design process, not just an

With Wavefront Differential Tolerancing, you can:

- Utilize the following performance metrics: RMS wavefront error, diffraction MTF, single mode fiber insertion loss or polarization-dependent loss, and Zernike wavefront coefficients.
- Perform a sensitivity analysis to the current tolerance set.
- Use the Inverse Sensitivity mode to automatically determine each tolerance within user-defined tolerance limits such that it contributes about equally to a specific system performance degradation for the worst case field and zoom position. A subset of tolerances can be *frozen* so that their values remain fixed.
- Use the Interactive Tolerancing mode to make changes to individual tolerance values and instantly see the performance impact.
- List tolerance sensitivities and performance predictions for every field and zoom position, with either common or independent compensation across field and zoom position.
- Assign specific compensators to specific tolerances by using tolerance and compensator labels.
- Force compensation based on field symmetry without requiring additional field points to be entered.
- Assign tolerances and compensators to a specific configuration of a multi-configuration lens (i.e., a specific zoom position).
- Override tolerance limits for each supported tolerance type.
- Create a new tolerance by grouping individual tolerances.
- Perform accurate tolerancing of double-pass systems or systems with parametric relationships among the constructional data (such as the front and rear radii and center thickness of a ball lens).
- Predict changes in distortion (or calibrated distortion) due to tolerances
- Compensate for image quality while simultaneously correcting line-of-sight errors (i.e., boresight correction) and magnification errors, due to tolerances.
- Define different tolerance probability distributions for different classes of tolerances.
- Identify the best compensator set from among all possible compensators using a singular value decomposition algorithm.

end-of-the-project analysis. The as-built performance of competing design forms or competing compensation approaches can be compared to determine the best system configuration that includes the impact of manufacturing and alignment. The tolerances that drive performance are immediately apparent.

Two Traditional Approaches to Tolerancing

Finite Difference and **Monte Carlo** are two common tolerancing algorithms. The Finite Difference approach individually varies each parameter within its tolerance range and analyzes the resulting system performance for each tolerance. These individual results are statistically combined to yield a total system performance prediction. This method predicts performance sensitivity for each tolerance, which helps to identify the individual parameters that are "performance drivers." To keep unnecessary cost out of a design, it is important to have tight tolerances only on those parameters that cause the greatest performance degradation for small changes. Only the most sensitive components should warrant the extra cost associated with tight tolerances.

The Finite Difference method does not consider how simultaneous changes in multiple parameters interact; its prediction of overall performance is typically optimistic. The effects of multiple tolerance interactions on the system performance are known as *cross-terms*. The Finite Difference method also suffers from numerical precision issues when a tolerance change causes a small difference between two very large numbers.

The Monte Carlo approach varies all of the fabrication parameters by random amounts within each tolerance range and typically uses optimization to compensate (i.e. refocus) the system. This simulates the performance of a single production unit chosen at random. The analysis of this random unit constitutes a single Monte Carlo *trial*. This process is repeated many times with different random perturbations. An accurate statistical prediction of the probability of achieving a particular performance level is generated if many trials (typically 100 to 1000) are run. Because all of the parameters are being varied at the same time, the Monte Carlo method accurately accounts for cross-terms. However, no information can be obtained from the Monte Carlo analysis about individual tolerance sensitivities. As such, you can accurately predict a system's as-built performance, but you cannot determine the specific tolerances that are driving the performance, and therefore cannot select the best set of tolerances to minimize cost.

Both the Finite Difference and Monte Carlo tolerancing methods are computationally intensive, which can be slow. With the Finite Difference method, a system's performance must be analyzed twice for each tolerance parameter, as both the plus and minus perturbations must be considered. Thus, more complex systems will take longer for a tolerance analysis than simpler systems. For example, a triplet typically has over 50 tolerances, resulting in over 100 required simulations.

Some optical design software packages utilize polynomial curve-fitting routines during the initial Finite Difference tolerance analysis to decrease the computational time required for subsequent tolerance analyses. In this case, the effect of changing a tolerance value can be quickly analyzed using the polynomial coefficients. However, this approach is useful only if tolerancing is the last step of the design; otherwise, the polynomials will need to be recalculated every time the design changes, adding to the overall time required for both design and tolerancing.

In the Monte Carlo approach, the system must be analyzed for every trial. System complexity is less of an issue, but many trials are required to achieve an accurate performance prediction. Analyzing a complex system to a reasonable level of accuracy using either the Finite Difference or the Monte Carlo method may require many hours (or even days) of analysis time.

Wavefront Differential Tolerancing

The Wavefront Differential algorithm is very fast and combines the best attributes of both the Finite Difference and Monte Carlo methods. The Wavefront Differential method provides information about individual tolerance sensitivities (like the Finite Difference method) and a more accurate performance prediction, including the effect of cross-terms (like the Monte Carlo method). For tolerances that cause a small change to the overall performance, the Wavefront Differential method is also more accurate than the Finite Difference method.

The speed of the Wavefront Differential approach is derived from the design of the algorithm. All of the information needed for the initial and all subsequent tolerance analyses is obtained from the nominal system by tracing a single group of rays. This single-pass approach is extremely fast, even when compared to curve-fitting routines.

The algorithmic foundation for the Wavefront Differential analysis method is based on the work of Hopkins & Tiziani[1], King[2], and Optical Research Associates' Chief Scientist, Matthew Rimmer[3,4]. The advanced algorithms developed by Mr. Rimmer used in CODE V's tolerancing feature (**TOR**) were first implemented in CODE V in 1978, decades prior to any other commercial implementation. The CODE V Wavefront Differential algorithms have been continually enhanced since they were first introduced, and include many proprietary features and advanced capabilities not found in any other software package.

Assumptions of the Wavefront Differential Method

The accuracy of the Wavefront Differential method is dependent on a few assumptions. Primarily, the ray optical path differences (OPDs) due to tolerance perturbations are assumed to vary linearly with the perturbation. Typically, this assumption is valid if the perturbation is small and results in a slight degradation of the nominal performance, which is what a designer nominally tries to achieve when tolerancing a system.

Also, the Wavefront Differential method is only applicable to performance metrics that can be computed by analyzing the complex field at the exit pupil of the system. Such metrics include wavefront error, diffraction MTF, fiber coupling insertion loss, polarization-dependent insertion loss, and Zernike wavefront coefficients. Additionally, development of the Wavefront Differential equations requires knowledge of how each tolerance affects the system; CODE V's Wavefront Differential tolerancing option (**TOR**) will analyze pre-programmed (i.e., built-in) tolerance types, of which there are over 50. Also, new tolerance types can be "synthesized" by combining existing tolerances (e.g., a ball lens diameter tolerance can be constructed from two radii and one thickness tolerance).

Finally, Wavefront Differential tolerancing assumes that the overall performance probability has a Gaussian form, defined by a mean and sigma. This assumption is typically valid if each tolerance is contributing about the same to the overall performance degradation, which is what **TOR**'s inverse sensitivity analysis tries to achieve. Otherwise, the Gaussian probability assumption tends to be conservative. It is important to understand that the Wavefront Differential method includes cross-terms; wavefront differentials are computed for each tolerance and for every pair of tolerances, so these important factors are included in the overall predicted performance for the system.

The fast Wavefront Differential method can be effectively used in concert with other tolerancing methods, for optical engineers who are accustomed to alternative algorithms. Designers often take advantage of the speed of the Wavefront Differential method's inverse sensitivity analysis to quickly determine appropriate tolerances and compensators. Afterward, a single Monte Carlo analysis of the resulting system (consisting of a large number of trials) can provide assurance of the accuracy of the Wavefront Differential performance predic-

Table 1: Feature summary of CODE V's primary tolerancing methods

Tolerancing Algorithm	CODE V Command	Supported Performance Metrics	Supported Tolerances	Comments
Wavefront Differential	TOR	• RMS Wavefront Error • Diffraction MTF • Fiber Coupling Efficiency • Polarization Dependent Loss • Zernike Coefficients-based error function	CODE V pre-programmed tolerances (e.g., radius, thickness, wedge, barrel tilt, test plate fit (in fringes), etc.)	• Very fast • Very accurate for tolerances that result in a small degradation in system performance (includes cross-terms) • Provides individual tolerance sensitivities *and* accurate performance prediction • Both inverse sensitivity and sensitivity analyses are supported • Supports optional distortion analysis, boresight correction, and an SVD compensator algorithm
Finite Difference	TOLFDIF (macro)	Any quantity that CODE V can compute	Most CODE V pre-programmed tolerances & macro-based *user-defined* tolerances	• Can be slow depending on number of tolerances, fields, zooms, and type of performance metric analyzed • Provides accurate individual tolerance sensitivities, particularly for larger tolerances • Performance summary is optimistic because cross-terms are not included • Method assumes that the performance variation is quadratic with tolerance, which may not be valid for the requested performance metric • Sensitivity analysis only
Monte Carlo	TOLMONTE (macro)	Any quantity that CODE V can compute	Most CODE V pre-programmed tolerances & macro-based *user-defined* tolerances	• Can be slow depending on the number of trials requested and type of performance metric analyzed • Provides accurate performance prediction (if many trials are requested) • No information about individual tolerance sensitivities • Sensitivity analysis only

tion. After gaining experience with the Wavefront Differential method, most users find that the extra Monte Carlo analysis step is unnecessary.

CODE V fully supports the Finite Difference and Monte Carlo tolerancing methods for systems in which the Wavelength Differential method's assumptions are invalid. Table 1 lists the supported performance metrics and tolerance types for each of the three methods.

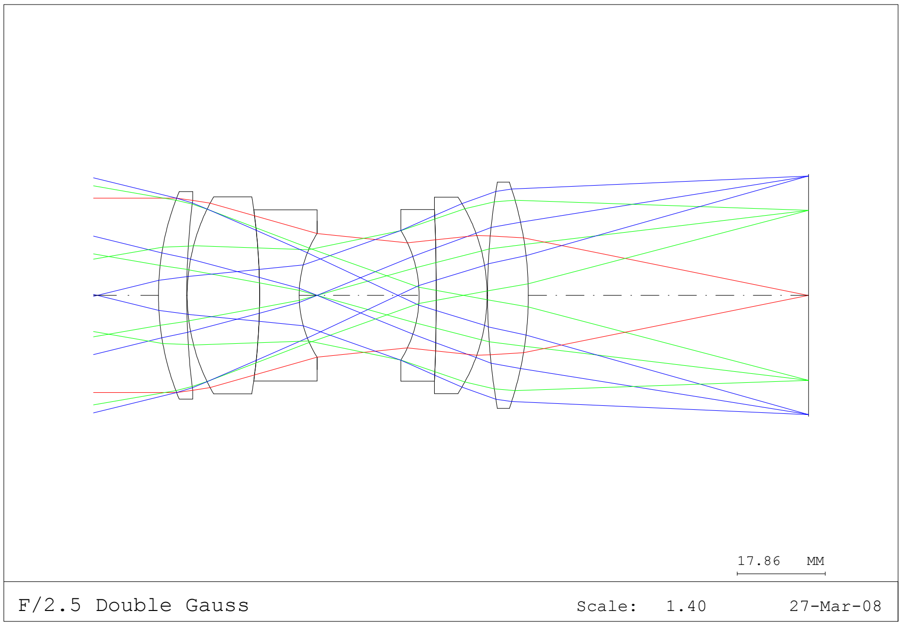
The applicability of CODE V's Wavefront Differential tolerancing method to real systems is affirmed by its successful use within the ORA Engineering Services Group to define tolerances and analyze as-built performance for over a thousand fabricated designs. Additionally, thousands of systems have been successfully analyzed and fabricated by CODE V customers around the world; Wavefront Differential tolerancing can be a powerful feature in your optical design toolkit.

Example System: F/2.5 Double Gauss Objective

To illustrate these different tolerancing methods, consider an F/2.5, 14° half-FOV Double Gauss lens, as shown in Figure 1. The comparison uses the default set of CODE V tolerances which, for this lens system, is 50 centered tolerances (i.e., thickness, index, power, or irregularity) and 16 decentered tolerances (i.e., wedge, element tilt, or element decenter). The tangential MTF at 15 cycles/mm is used as the performance metric for the tolerance analysis, and the only allowed compensation is a longitudinal shift of the image plane. This type of compensator is often called *refocus*.

Table 2: Speed comparison of tolerancing methods

Tolerancing Method	Computation Time for a Pentium® M 1.7GHz CPU
Wavefront Differential (TOR)	3 seconds
Finite Difference (TOLFDIF)	58 seconds (19x TOR)
Monte Carlo - 1000 trials (TOLMONTE)	41 minutes, 12 seconds (824x TOR)



F/2.5 Double Gauss Lens

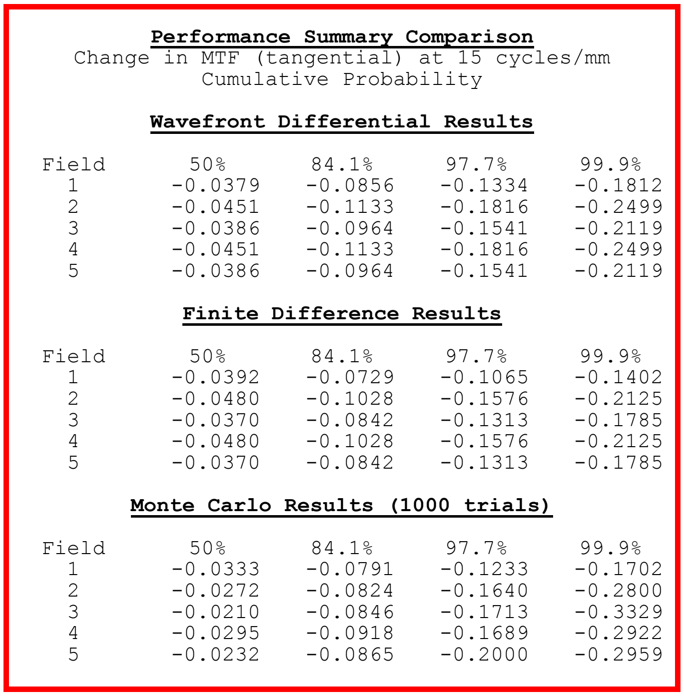
For this analysis all scalar and cylindrical parameters are assumed to have an equal probability of having any value within the plus and minus tolerance limits. Two-dimensional decentered tolerances use a Gaussian probability distribution. Additionally, the tolerance probability distribution can be modified in CODE V for different classes of tolerances. The system will be analyzed for the following five field positions: on-axis, +/-70% field, and +/- Full Field. Symmetric fields are used to show symmetry in the tolerance methods.

Using these settings, three tolerance analyses were performed using the described algorithms. Table 2 compares the relative speed of the tolerancing methods. The time required to fully tolerance the system using the Wavefront Differential method is approximately equivalent to running a single trial of the Monte Carlo method. This relationship makes sense because each trial of Monte Carlo method requires a ray trace of the system, which is all that

Table 3: Single tolerance result comparison of Wavefront Differential and Finite Difference methods

```
                 Single Tolerance Comparison

          (Delta Radius of surface 5, ± 0.025)

               Wavefront Differential Results

        Field            Change in MTF (tangential)
                               at 15 cycles/mm

                           - Tolerance    + Tolerance
   1 (On-axis)              -0.0166         0.0132
   2 (+10 deg)              -0.0074         0.0011
   3 (+14 deg)               0.0063        -0.0092
   4 (-10 deg)              -0.0074         0.0011
   5 (-14 deg)               0.0063        -0.0092

 Compensator (refocus) Motion =   -0.116363
 2Σ Comp. Motion for all tol. =   ±0.548248

                 Finite Difference Results

        Field            Change in MTF (tangential)
                               at 15 cycles/mm

                           - Tolerance    + Tolerance
   1 (On-axis)              -0.0125         0.0114
   2 (+10 deg)              -0.0111         0.0040
   3 (+14 deg)              -0.0088        -0.0115
   4 (-10 deg)              -0.0111         0.0040
   5 (-14 deg)               0.0088        -0.0115

 Compensator (refocus) Motion =   -0.118785
 2Σ Comp. Motion for all tol. =   ±0.550359
```

Table 4: Cumulative performance summary for three tolerancing methods



```
                Performance Summary Comparison
          Change in MTF (tangential) at 15 cycles/mm
                    Cumulative Probability

                Wavefront Differential Results

 Field        50%        84.1%       97.7%       99.9%
   1       -0.0379     -0.0856     -0.1334     -0.1812
   2       -0.0451     -0.1133     -0.1816     -0.2499
   3       -0.0386     -0.0964     -0.1541     -0.2119
   4       -0.0451     -0.1133     -0.1816     -0.2499
   5       -0.0386     -0.0964     -0.1541     -0.2119

                  Finite Difference Results

 Field        50%        84.1%       97.7%       99.9%
   1       -0.0392     -0.0729     -0.1065     -0.1402
   2       -0.0480     -0.1028     -0.1576     -0.2125
   3       -0.0370     -0.0842     -0.1313     -0.1785
   4       -0.0480     -0.1028     -0.1576     -0.2125
   5       -0.0370     -0.0842     -0.1313     -0.1785

              Monte Carlo Results (1000 trials)

 Field        50%        84.1%       97.7%       99.9%
   1       -0.0333     -0.0791     -0.1233     -0.1702
   2       -0.0272     -0.0824     -0.1640     -0.2800
   3       -0.0210     -0.0846     -0.1713     -0.3329
   4       -0.0295     -0.0918     -0.1689     -0.2922
   5       -0.0232     -0.0865     -0.2000     -0.2959
```

the Wavefront Differential method requires for its complete analysis.

The Wavefront Differential and Finite Difference tolerancing methods provide information about individual tolerance sensitivities. Table 3 shows the change in performance that results from a perturbation of the radius of surface 5 for both methods. Notice that the predicted compensation motion, which is directly calculated with the Wavefront Differential method and solved via optimization in the Finite Difference method, correlates well between the two methods. Also, the predicted compensator motion range for all tolerances correlate very well for 2Σ, or approximately 98%, of all systems.

Table 4 compares the cumulative probability performance summary for the three methods. It lists the MTF degradation at different probability levels. The values in the table represent maximum degradation in MTF for a given field for a certain percentage of built systems. The results correlate well, especially on-axis and at the lower performance probabilities off-axis. The Finite Difference prediction is somewhat optimistic because it does not include cross-terms. The correlation between the Wavefront Differential method and the Monte Carlo method does diverge at the higher probabilities for the off-axis points. These differences can be understood by examining the cumulative probability performance plots.

Each of three CODE V tolerancing methods discussed in this paper can create a cumulative probability performance plot. These plots show the performance at any probability level. Figure 2 is the cumulative probability plot generated by the Wavefront Differential tolerancing method (**TOR**). Figure 3 is the cumulative probability plot generated by the Monte Carlo method (**TOLMONTE**). At the higher probabilities, the Monte Carlo curves have significant "tails." Also observe in Table 4 that the Monte Carlo results for symmetric fields are not the same. These two observations suggest that more trials may be required to achieve, with Monte Carlo, the level of accuracy that exists in the Wavefront Differential result.

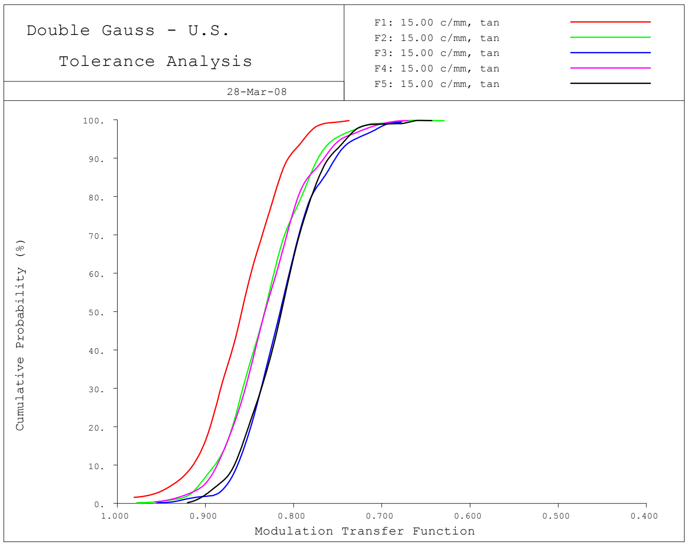


Figure 2: Wavefront Differential cumulative probability plots

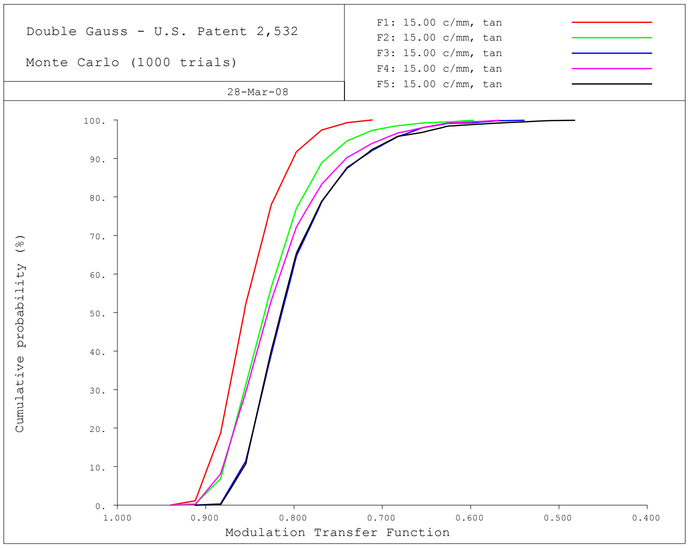


Figure 3: Monte Carlo cumulative probability plot

CODE V's optimization capabilities have long been recognized as the best in the industry for achieving optimum nominal performance with minimized system complexity. However, for systems destined for fabrication, outstanding nominal performance is only the first step to a successful product.

This paper demonstrates how CODE V's advanced tolerancing features provide outstanding speed, accuracy, and flexibility, which ultimately help to maintain optical system performance while reducing costs during product development and throughout the product life cycle.

References

1 H.H. Hopkins and H.J. Tiziani. “A theoretical and experimental study of lens centring errors and their influence on optical image quality.” *Brit. J. Appl. Phys*. Vol. 17. 33 (1966).

2 W. B. King. “Use of the Modulation-Transfer Function (MTF) as an Aberration-Balancing Merit Function in Automatic Lens Design.” *JOSA*. Vol 59. 1155 (1969).

3 M. Rimmer. “Analysis of Perturbed Lens Systems.” *Appl. Opt*., Vol 9, 533 (1970).

4 M. Rimmer. “A Tolerancing Procedure Based on Modulation Transfer Function (MTF).” *SPIE*. Vol. 147, 66 (1978).

About ORA

Optical Research Associates (ORA®) is the industry’s leading supplier of imaging and illumination design/analysis software: CODE V® and *LightTools*®. Our Engineering Services group is the largest independent supplier of optical systems design with more than 4,500 completed projects in imaging, illumination, and optical systems engineering. ORA was founded in 1963 to provide leading-edge optical design services. ORA’s primary vision – to accelerate the development and adoption of optical technology throughout the world – has led to its definitive role as an innovative solutions supplier to the optics industry.

OPTICAL
RESEARCH
ASSOCIATES

3280 East Foothill Boulevard, Pasadena, California 91107
Telephone: (626) 795-9101 FAX: (626) 795-0184
E-mail: service@opticalres.com Web site: http://www.opticalres.com