



CODE V Tolerancing: A Key to Product Cost Reduction

A critical step in the design of an optical system destined to be manufactured is to define a fabrication and assembly tolerance budget and to accurately predict the resulting as-built performance, including the effects of compensation (e.g., refocus). In addition, determining the best set of compensators from among several choices can have a significant impact on both the recurring and non-recurring costs. This complex process is often simply called, “tolerancing.”

CODE V is a comprehensive software package for the design, analysis, tolerancing, and fabrication support of optical systems. It is one of two advanced optical engineering software packages offered by Optical Research Associates (ORA®). CODE V is used by organizations around the world to design a wide range of optical systems for a variety of products, including photographic equipment, medical instruments, aerospace systems, telecommunication components, microlithographic stepper systems, and much more. Many of the companies and users who have chosen CODE V do so for its advanced algorithms and features related to tolerancing and optimization.

Why Is Tolerancing Important?

In a word, **cost**. Many optical design software packages will do a reasonable job of optimizing a design to have good as-designed performance. However, if small variations in the values of the lens parameters result in significant loss of performance even after compensation is applied, the cost to build the design can be prohibitively high. To minimize production costs, the ideal optical system design will maintain

the required performance with achievable component and assembly tolerances, using well-chosen post-assembly adjustments. CODE V’s unique suite of tolerancing capabilities can help to make this ideal system a reality.

What Makes Tolerancing with CODE V So Successful?

In two words, **speed** and **accuracy**. Many optical design software packages have features for

tolerancing, but the algorithmic approach often requires extremely long computation times to achieve accurate results. CODE V includes a **Wavefront Differential** tolerancing method that is extremely fast and accurate. Using this method, tolerancing becomes part of the design process, not just an end-of-the-project analysis. In addition, it becomes practical to compare the as-built performance of competing design forms or competing compensation approaches to determine the best system configuration that includes the impact of manufacturing and alignment issues.

Features of CODE V’s Wavefront Differential Tolerancing

The significant features of CODE V’s **TOR** option include:

- Performance metrics of RMS wavefront error, diffraction MTF, and single mode fiber insertion loss (with polarization-dependent loss available soon).
- An inverse sensitivity mode, where CODE V determines each tolerance value within user-defined limits such that each tolerance contributes about equally to the system performance degradation for the worst case field and zoom position.
 - A sensitivity analysis to the current tolerance set is also supported, along with the ability to do an inverse sensitivity analysis with a subset of tolerances “frozen” so that their value remains fixed.
- Tolerance sensitivities and performance predictions listed for every field and zoom, with either common or independent compensation across field and zoom.
- Tolerance and compensator labels to assign specific compensators to specific tolerances.
- The ability to force compensation based on field symmetry without requiring additional field points to be entered.
- The ability to assign tolerances and compensators to a specific configuration of a multi-configuration lens (i.e., a specific “zoom” position).
- The ability to override tolerance limits for each supported tolerance type.
- The ability to create a new tolerance by grouping individual tolerances.
 - Allows accurate tolerancing of double-pass systems or systems with parametric relationships among the constructional data (such as the front and rear radii and center thickness of a ball lens).
- The ability to predict changes in distortion (or calibrated distortion) due to tolerances
- The ability to compensate for image quality while simultaneously correcting line-of-sight errors (i.e., boresight correction) and magnification errors, due to tolerances.
- The ability to define different tolerance probability distributions for different classes of tolerances.

Additional features and capabilities will be added to future releases.

Two Traditional Approaches to Tolerancing

To understand the advantages of CODE V's tolerancing method, it is worthwhile to review two traditional methods, **Finite Differences** and **Monte Carlo** analyses.

The Finite Differences approach individually varies each parameter within its tolerance range and predicts the system performance degradation on a tolerance-by-tolerance basis. These individual results are statistically combined to yield a total system performance prediction. This method accurately predicts performance sensitivity to individual tolerances, which allows determination of the parameters that are "performance drivers." However, since the Finite Differences method does not consider how simultaneous parameter changes by multiple tolerances will interact, its prediction of overall performance is typically optimistic. The effects of tolerance interactions on the system performance are known as "cross-terms."

The Monte Carlo approach is to vary all of the parameters that have an associated tolerance by random amounts, but within each tolerance range. The resulting system performance is analyzed. This process is repeated many times with different random perturbations (each analysis is often referred to as a "trial"). If many trials are run (100 to 1000 is typical), an accurate statistical prediction of the probability of achieving a particular performance level can be generated. Since all the parameters are being varied at the same time, the Monte Carlo method accurately accounts for cross-terms. However, no information can be gleaned from the Monte Carlo analysis about individual tolerance sensitivities. Therefore, while you can accurately predict system as-built performance, you cannot determine the significant parameters that are driving the performance, and thus cannot select the best set of tolerances to minimize cost.

Both the Finite Differences and Monte Carlo tolerancing methods are very computationally intensive and can be very slow. For Finite Differences, the system must be analyzed twice, once for each tolerance (i.e., the plus and minus perturbation). Thus, more complex systems will take longer to "tolerance" than simpler systems. A triplet typically has over 50 tolerances, resulting in over 100 analysis simulations. For the Monte Carlo approach, the system must be analyzed for every trial. System complexity is less of an issue, but the accuracy of the performance prediction increases as the number of trials is increased. A hundred to a thousand analysis simulations are typical. Analyzing a complex system to high accuracy using both the Finite Differences and Monte Carlo methods can require many hours (or even days) of analysis time.

CODE V Tolerancing Capabilities

While CODE V supports both the Finite Differences and Monte Carlo methods, the primary tolerance analysis feature of CODE V uses a unique Wavefront Differential algorithm that is very fast, and provides information about both individual tolerance sensitivities (like the Finite Differences method) and an accurate performance prediction, including the effect of cross-terms (like the Monte Carlo method). For tolerances that cause a small change to the overall performance, the wavefront differential method can also be more accurate than Finite Differences, which can suffer numerical precision problems when subtracting two large performance numbers to determine a small difference.

The reason that the Wavefront Differential approach is so fast compared to either the Finite Differences or Monte Carlo methodologies is that the nominal system is ray traced once, and all the required information for further analysis is extracted by CODE V algorithms from this ray trace of the nominal system.

The algorithmic foundation for the Wavefront Differentials analysis method is based on the work of Hopkins & Tiziani¹, King² and Optical Research Associates' Chief Scientist, Matthew Rimmer^{3,4}. The detailed algorithms developed by Mr. Rimmer are used in CODE V's tolerancing feature (**TOR**), MTF optimization feature, and automatic alignment feature (**ALI**). They were first implemented in CODE V in 1978, decades prior to any other commercial implementation. The CODE V Wavefront Differential algorithms have been continually enhanced since they were first introduced, and include many proprietary features and advanced capabilities not found in any other software package.

Assumptions of the Wavefront Differential Method

The accuracy of the Wavefront Differential method is subject to a few assumptions. The primary assumption is that ray optical path differences (OPDs) due to tolerance perturbations vary linearly with tolerance change. This assumption is typically valid if the tolerance perturbation results in a small degradation of the nominal performance. This is in fact what the designer typically tries to achieve when tolerancing a system. Also, the Wavefront Differential method is only applicable to performance metrics that can be computed by analyzing the complex field at the exit pupil of the system. The metrics currently implemented in CODE V are RMS wavefront error, diffraction MTF, and fiber coupling insertion loss (polarization-dependent insertion loss will be added soon). Additionally, development of the Wavefront Differential equations requires knowledge of how each tolerance affects the system. This means that CODE V's **TOR** option will only analyze pre-programmed (i.e., built-in) tolerance types. A final assumption of Wavefront Differential tolerancing method implemented in CODE V is that the overall performance probability has a Gaussian form, defined by a mean and sigma. This assumption is typically

valid if each tolerance is contributing about the same to the overall performance degradation (exactly what the inverse sensitivity mode of **TOR** tries to achieve). When this is not the case, the Gaussian probability assumption tends to be conservative. It is important to understand that CODE V's **TOR** option does include cross-terms. Wavefront differentials are computed for each individual tolerance and for every pair of tolerances, so these important factors are included in the overall predicted performance for the system.

For optical engineers who are accustomed to alternative tolerance methods such as finite differences or Monte Carlo, the fast wavefront differential method can be effectively used in concert with other methods. Users often take advantage of the speed of the Wavefront Differential

method's inverse sensitivity analysis to quickly determine appropriate tolerances and compensator(s). A single Monte Carlo analysis of the resulting system (consisting of a large number of trials) can provide assurance of the accuracy of the wavefront differential performance prediction. After gaining experience with the Wavefront Differential method, most users will find that the extra Monte Carlo analysis step is unnecessary.

The applicability of CODE V's Wavefront Differential tolerancing method to real system tolerancing is borne out by the ORA Engineering Services Group's successful use of the **TOR** option to define tolerances and analyze as-built performance for over a thousand fabricated designs. Add to this the additional thousands of systems successfully analyzed and

fabricated by CODE V customers around the world, and you realize that **TOR** can be a powerful feature in your optical design toolkit as well.

The Three Tolerancing Methods Compared

For those cases where CODE V's Wavefront Differential method does not support the desired performance metric, where the tolerances are large such that ray OPDs do not vary linearly with tolerance change, or where tolerances are desired that are not currently pre-defined in CODE V, the program also supports Finite Differences and Monte Carlo tolerancing.

Table 1 compares CODE V's different tolerancing methods.

Table 1. CODE V's tolerancing methods

Algorithm	CODE V Feature	Supported Performance Metric	Supported Tolerances	Comments
Wavefront Differentials	TOR	<ul style="list-style-type: none"> RMS Wavefront Error Diffraction MTF Fiber Coupling Efficiency into a SMF Polarization Dependent Loss into a SMF 	CODE V pre-programmed tolerances (e.g., DLR , DLT , TIR , BTI , etc.)	<ul style="list-style-type: none"> Very fast Very accurate for tolerances that result in a small degradation in system performance (includes cross-terms) Provides individual tolerance sensitivities AND accurate performance prediction Both Inverse Sensitivity & Sensitivity analysis supported
Finite Differences	TOLFDIF	Any quantity that CODE V can compute	CODE V pre-programmed tolerances & <i>User-defined tolerances</i>	<ul style="list-style-type: none"> Can be slow depending on number of tolerances, fields, zooms and type of performance metric analyzed. Provides accurate individual tolerance sensitivities, particularly for larger tolerances Performance summary is optimistic since this method does not include cross-terms. Performance summary is approximate since this method assumes that the performance variation is quadratic with tolerance. This assumption may not be valid for the requested performance metric
Monte Carlo Simulation	TOLMONTE	Any quantity that CODE V can compute	CODE V pre-programmed tolerances & <i>User-defined tolerances</i>	<ul style="list-style-type: none"> Can be slow depending on the number of trials requested and type of performance metric analyzed. Provides accurate performance prediction (if many trials are requested), but no information about individual tolerance sensitivities

Example System: F/2.5 Double Gauss Objective

Let's illustrate these different tolerancing methods on a common example. The example is an F/2.5 Double Gauss lens with a default set of CODE V tolerances (50 centered tolerances such as thickness, index, power and irregularity & 16 decentered tolerances such as wedge, element tilt and element decenter). See Figure 1.

The performance metric is the tangential MTF at 15 cycles/mm and the only allowed compensation is a longitudinal shift of the image plane (i.e., refocus). For this analysis all parameters are assumed to have an equal probability of having any value within the plus and minus tolerance limits. In actuality, the tolerance probability distribution can be modified in CODE V for different classes of tolerances. The system will be analyzed for five field positions (on-axis, +/-70% field, and +/- Full Field).

Before we discuss the results, it is noteworthy to compare the speed of the Wavefront Differential method relative to a Finite Differences or Monte Carlo method (Table 2).

The time required to fully tolerance the system using the Wavefront Differential method is approximately equivalent to running a single trial with Monte Carlo. This makes sense since each trial of Monte Carlo requires a ray trace of the system, and that is all that the Wavefront Differential method requires for its complete analysis.

Table 2. Speed comparison of tolerancing methods

Tolerancing Method	Computation Time for PIII 1GHz PC
Wavefront Differential (TOR)	6-seconds
Finite Difference (TOLFDIF)	6.7-minutes (67 x TOR)
Monte Carlo – 1000 trials (TOLMONTE)	3.0-hours (1800 x TOR)

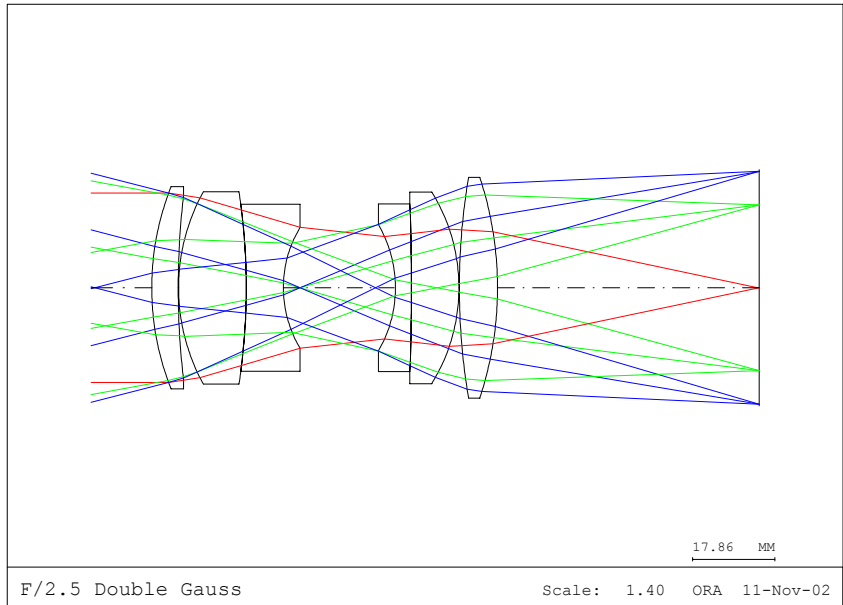


Figure 1. F/2.5 Double Gauss Lens

Finite Differences tolerancing provides information about individual tolerance sensitivities. Table 3 shows a comparison between the Wavefront Differentials and Finite Differences results for the change of radius tolerance on surface 5. Note the predicted compensation motion (included in the wavefront differential equations in TOR and via optimization in TOLFDIF) along with the predicted compensator motion range for all tolerances (to handle 98%, or 2-sigma of the systems). The results correlate very well.

Next, we compare the cumulative probability performance summary for the three methods. These are listed in terms of MTF

degradation at different probability levels. The way to read the table is that the MTF for a given field will have the indicated degradation or less for 50%, 84%, 98%, etc. of built systems (Table 4).

Table 3. Comparison of Wavefront Differentials and Finite Differences results

Single Tolerance Comparison			
(Delta Radius of surface 5, +/- 0.025)			
TOR Results			
Field	Change in MTF (tangential) at 15 cycles/mm		
	- Tol.	+ Tol.	
1 (On-axis)	-0.017	0.013	
2 (+10 deg)	-0.007	0.001	
3 (+14 deg)	0.006	-0.009	
4 (-10 deg)	-0.007	0.001	
5 (-14 deg)	0.006	-0.009	
Compensator (refocus) Motion = -0.116363			
2Σ Comp. Motion for all tol. = +/-0.5482			
TOLFDIF Results			
Field	Change in MTF (tangential) at 15 cycles/mm		
	- Tol.	+ Tol.	
1 (On-axis)	-0.013	0.011	
2 (+10 deg)	-0.011	0.005	
3 (+14 deg)	0.008	-0.011	
4 (-10 deg)	-0.012	0.005	
5 (-14 deg)	0.008	-0.011	
Compensator (refocus) Motion = -0.118785			
2Σ Comp. Motion for all tol. = +/-0.5476			

Table 4. Cumulative performance summary for three tolerancing methods

Performance Summary Comparison				
Change in TAN MTF at 15 cycle/mm				
CUMULATIVE PROBABILITY				
TOR Results				
Field	50.0%	84.1%	97.7%	99.9%
1	-0.047	-0.100	-0.153	-0.206
2	-0.062	-0.150	-0.238	-0.327
3	-0.053	-0.125	-0.198	-0.270
4	-0.062	-0.150	-0.238	-0.327
5	-0.053	-0.125	-0.198	-0.270
TOLFDIF Results				
1	-0.049	-0.083	-0.116	-0.150
2	-0.063	-0.127	-0.190	-0.254
3	-0.047	-0.099	-0.152	-0.204
4	-0.072	-0.136	-0.199	-0.263
5	-0.059	-0.113	-0.168	-0.223
TOLMONTE Results - 1000 trials				
1	-0.047	-0.103	-0.168	-0.237
2	-0.053	-0.134	-0.257	-0.380
3	-0.037	-0.129	-0.264	-0.422
4	-0.048	-0.143	-0.292	-0.477
5	-0.049	-0.150	-0.317	-0.422

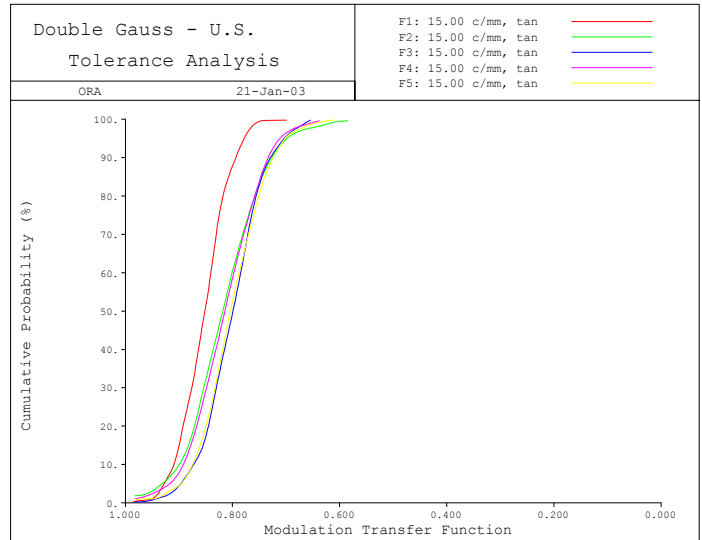


Figure 2. Wavefront Differentials cumulative probability plot

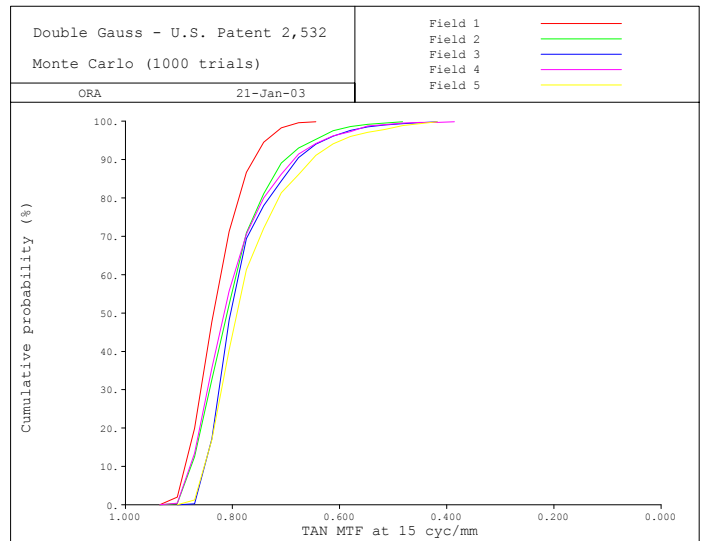


Figure 3. Monte Carlo cumulative probability plot

The results correlate well, especially on-axis and at the lower performance probabilities off-axis. The Finite Differences prediction is somewhat optimistic since it does not include cross-terms. The correlation between the Wavefront Differentials method and the Monte Carlo method does diverge at the higher probabilities for the off-axis points. These differences can be understood by examining the cumulative probability performance plots.

All three of the CODE V tolerancing methods discussed in this paper create a cumulative probability performance plot. This allows users to determine the performance at any probability level. Figure 2 is the cumulative probability plot generated by the Wavefront Differentials tolerancing method (TOR). Figure 3 is the cumulative probability plot generated by the Monte Carlo method (TOLMONTE). You can see that at the higher probabilities, the Monte Carlo curves have significant “tails.” This, and the fact that the Monte Carlo results are not symmetric for symmetric fields, suggest that more trials may be required for improved accuracy.

Conclusion

CODE V’s optimization capabilities have long been the recognized as the best in the industry for achieving optimum nominal performance with minimized system complexity.

However, for systems destined for fabrication, outstanding nominal performance is only the first step to a successful product.

This paper demonstrates how CODE V’s advanced tolerancing features provide outstanding speed, accuracy, and flexibility, which ultimately help you to maintain optical system performance while reducing costs during product development, and throughout the product life cycle.

References

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About ORA

Optical Research Associates (ORA[®]) is the industry's leading supplier of imaging and illumination design/analysis software: CODE V[®] and *LightTools*[®]. Our Engineering Services group is the largest independent supplier of optical systems design with more than 4,500 completed projects in imaging, illumination, and optical systems engineering. ORA was founded in 1963 to provide leading-edge optical design services. ORA's primary vision — to accelerate the development and adoption of optical technology throughout the world — has led to its definitive role as an innovative solutions supplier to the optics industry.

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